$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ 1st derivative

$$f''(x) = \lim_{h \to 0} \frac{f'(x+h) - f'(x)}{h}$$ 2nd derivative

$$f'''(x) = \quad - \quad -$$

$$f^{(4)}(x) = \lim_{h \to 0} \frac{f'''(x+h) - f'''(x)}{h}$$

$$\lim_{x_1 \to x_2} \frac{f(x_1) - f(x_2)}{x_1 - x_2} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

$$\frac{d}{dx}(y) = \frac{dy}{dx}, \quad \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$$

the derivative operator

$$\frac{d^3y}{dx^3}, \quad \frac{d^4y}{dx^2}, \quad \ldots$$

Ex: $f(x) = 2x^2 + x$

$$f'(x) = \lim_{t \to x} \frac{(2t^2+t) - (2x^2+x)}{t-x}$$

$$= \lim_{t \to x} \frac{(2t^2-2x^2) + (t-x)}{t-x}$$

$$= \lim_{t \to x} \frac{2(t+x)\underline{(t-x)} + (t-x)}{t-x}$$

$$= \lim_{t \to x} \frac{\cancel{(t-x)}\left(2(t+x)+1\right)}{\cancel{t-x}}$$

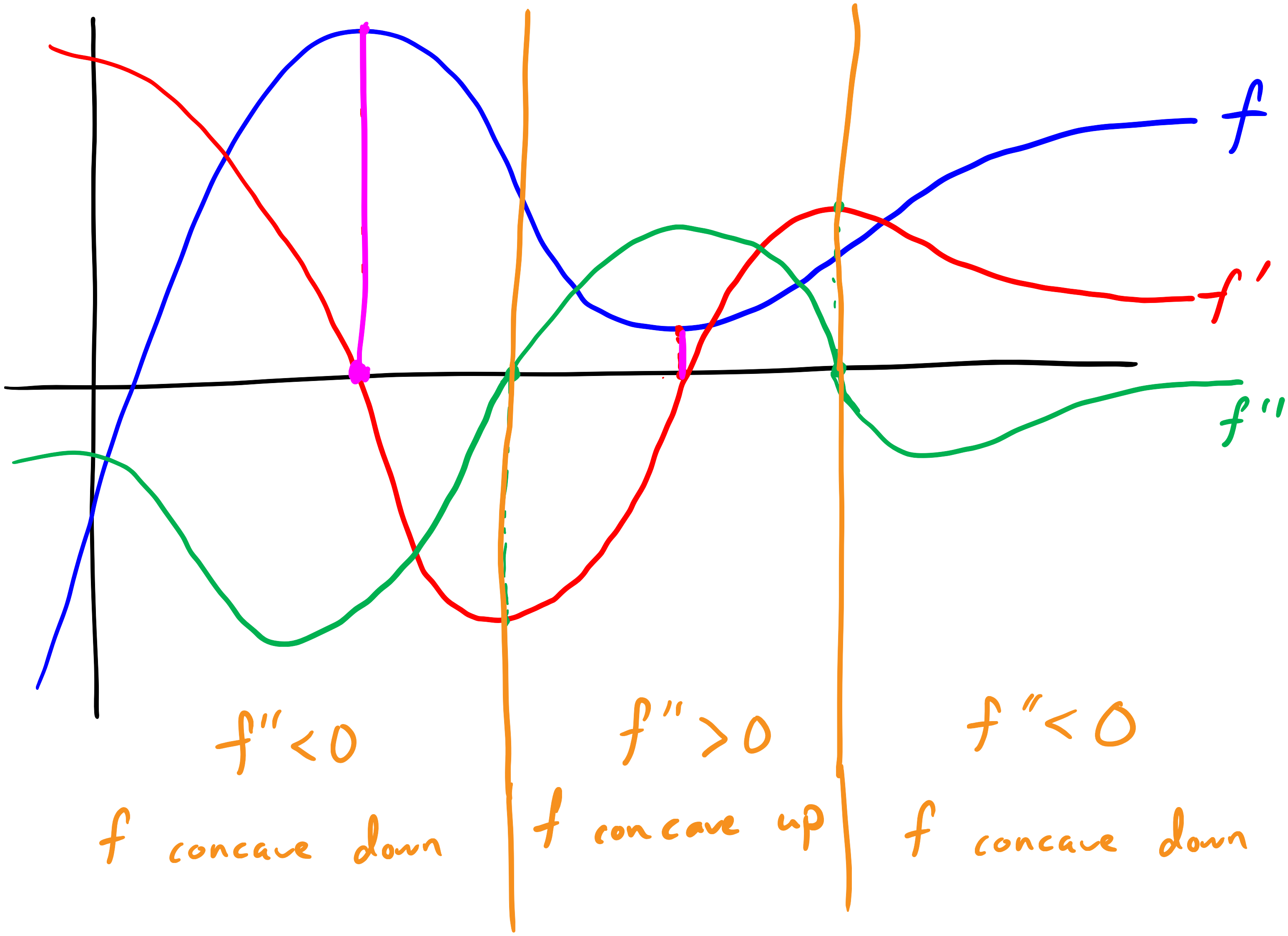
$$= \lim_{t \to x} (2t+2x+1) = 2x+2x+1$$

$$= 4x+1$$

$$f''(x) = \lim_{h \to 0} \frac{f'(x+h) - f'(x)}{h}$$

$$= \lim_{h \to 0} \frac{(4(x+h)+1) - (4x+1)}{h} = \lim_{h \to 0} \frac{4h}{h}$$

$$= \lim_{h \to 0} 4 = 4$$



Physical meaning for derivatives

$s(t)$ - position function

$s'(t) = v(t)$ - velocity function

$s''(t) = v'(t) = a(t)$ - acceleration function

$s'''(t) = v''(t) = a'(t) = j(t)$ - jerk function

Rules for Differentiation

① $\frac{d}{dx}[c] = \lim_{t \to x} \frac{c-c}{t-x} = \lim_{t \to x} 0 = 0$

$$\boxed{\frac{d}{dx}[c] = 0}$$

$$\frac{d}{dx}[x] = \lim_{t \to x} \frac{t-x}{t-x} = \lim_{t \to x} 1 = 1$$

$$\frac{d}{dx}[x^2] = \lim_{t \to x} \frac{t^2-x^2}{t-x} = \lim_{t \to x} \frac{\cancel{(t-x)}(t+x)}{\cancel{t-x}}$$

$$= \lim_{t \to x} (t+x) = x+x = 2x$$

$$\frac{d}{dx}[x^3] = \lim_{t \to x} \frac{t^3-x^3}{t-x} = \lim_{t \to x} \frac{\cancel{(t-x)}(t^2+tx+x^2)}{\cancel{t-x}}$$

$$= \lim_{t \to x} (t^2+tx+x^2) = x^2+x^2+x^2 = 3x^2$$

$$\frac{d}{dx}[x^4] = \lim_{t \to x} \frac{t^4-x^4}{t-x} = \lim_{t \to x} \frac{\cancel{(t-x)}(t^3+t^2x+tx^2+x^3)}{\cancel{t-x}}$$

$$= \lim_{t \to x} (t^3+t^2x+tx^2+x^3) = 4x^3$$

② $\frac{d}{dx}[x^n] = nx^{n-1}$ **Power Rule**

<u>proof</u>:

$$\frac{d}{dx}[x^n] = \lim_{t\to x} \frac{t^n - x^n}{t-x}$$

$$= \lim_{t\to x} \frac{\cancel{(t-x)}\left(t^{n-1} + t^{n-2}x + t^{n-3}x^2 + \cdots + tx^{n-2} + x^{n-1}\right)}{\cancel{t-x}}$$

$$= \lim_{t\to x}\left(\overset{1}{t^{n-1}} + \overset{2}{t^{n-2}x} + \overset{3}{t^{n-3}x^2} + \cdots + \overset{n-1}{tx^{n-2}} + \overset{n}{x^{n-1}}\right)$$

$$= \underbrace{x^{n-1} + x^{n-1} + x^{n-1} + \cdots + x^{n-1} + x^{n-1}}_{n\text{-total}}$$

$$= n \cdot x^{n-1}$$